Using Fuzzy Intervals to Represent Measurement Error and Scientific Uncertainty in Endangered Species Classification

Scott Ferson, H. Resit Akçakaya and Amy Dunham
Applied Biomathematics
100 North Country Road
Setauket, New York 11733 USA
redlist@ramas.com

Abstract

Although fuzzy numbers (including fuzzy intervals) are often used to capture semantic ambiguity, they are also useful to represent and propagate measurement error. In this application, a classification scheme used by international authorities for assigning biological species into categories of relative endangerment is generalized to accept intervals and triangular or trapezoidal fuzzy numbers as inputs representing empirical estimates of unknown quantities. Non-traditional definitions for fuzzy magnitude comparisons and logical operations were required but, otherwise, standard fuzzy arithmetic was used. A defuzzification step, which explicitly reveals the analyst's attitudes regarding evidence, can condense the result from the fuzzified classification scheme to a single category. But this step is not required and may be counterproductive.

Introduction

The classification of biological species into categories of endangerment (critical, endangered, vulnerable, etc.) is essential for planning effective conservation strategies. The IUCN (World Conservation Union), a partner of the United Nations Environment Programme, has promulgated a comprehensive system [1] for making these classifications based on available information about a species' abundance, spatial distribution and life history. The IUCN classification system has been very widely used around the world over the last decade to classify thousands of species and to make sensitive management and regulatory decisions. Such decision rules are attractive because of their wide applicability, objectivity, and simplicity of use [2]. Although the IUCN scheme is but one of several that have been suggested by conservation biologists [3–6], it alone has enjoyed broad international acceptance, and it has become one of the most important decision tools in conservation biology.

The IUCN classification scheme has two conceptual problems. The first (which will be addressed in a separate contribution by H. Regan et al. [7]) is that the categories are defined in terms of precise scalar thresholds whose specificity has little biological justification. The second, addressed in this paper, is that the classification presumes that analysts can make precise numerical estimates for several important variables, even for extremely rare and poorly studied species. This is unlikely to be possible, even for well studied species, and calls into question the reliability of the resulting classification.

Measurement error and scientific uncertainty is ubiquitous throughout the empirical sciences, but they can be particularly large in the life sciences, especially ecology, because of the importance of stochasticity and complex, often nonlinear dynamics in natural ecosystems. For this reason, it seemed clear that the IUCN criteria should be generalized to account for measurement error. We wanted a robust approach that could be intelligible and useful to biologists without special training in uncertainty propagation techniques. We considered several possible approaches, including Monte Carlo uncertainty propagation [8], probability bounds analysis [9], interval analysis [10] and fuzzy arithmetic [11]. The most reasonable approach appeared to be one based on fuzzy arithmetic.

The IUCN rules for classification of threatened species consist of three sets. These sets define the classes Critically Endangered (CR), Endangered (EN), and Vulnerable (VU). The evaluation of a given taxon under each of these sets of rules results in a Boolean (TRUE or FALSE) outcome and the rule sets are nested in such a way that there are only four classifications possible: CR, EN, VU and a none-of-the-above classification denoted Lower Risk (LR). Therefore, if a taxon fires the CR rule, it will have also fired the EN and VU rules. A taxon is LR if it is in none of the other three categories. Each rule was defined by the IUCN as a Boolean disjunction with five components

A OR B OR C OR D OR E

Each of these five components is defined in terms of comparisons between certain life history traits for the taxon and threshold values. For example, Criterion D asks whether the number of mature individuals extant in the taxon is larger than some threshold value. Likewise, Criterion E asks whether the estimated probability of extinction is larger than some threshold. These thresholds are different but nested for the three rule sets. Some criteria consist of compound comparisons joined together with the logical operators AND and OR. For example, Criterion A for CR can be summarized as:

(Past reduction $\geq$ 80%) OR (Future reduction $\geq$ 80%)

where both past and future reduction are measured over a time period of 10 years or 3 generations, whichever is longer.

There are various ways of specifying uncertainty in a quantity such as the number of mature individuals. When natural variability is the main source of uncertainty and the sampling program has been sufficiently rigorous, we may specify confidence intervals. Most of the criteria in the IUCN rules include at least some semantic uncertainty. Even those variables amenable to measurement sometimes are not measured, but are the subject of expert opinion or guesswork informed by a few unrepresentative samples. One of the simplest ways to represent uncertainty when different kinds of uncertainty are confounded is to specify a best estimate and a range of plausible values. For example, we may represent an adult population size with a best estimate of 90 and bounds of 70 and 120, based on confidence intervals, the opinion of a single expert, or the consensus opinion of a group of experts. The resulting information can be represented as a triangular fuzzy number, graphed with the x-axis as the number of mature individuals and the y-axis as the possibility level. At each possibility level, there is an interval defined by the left and right sides of the fuzzy number. This level is an inverse measure of the surety that the parameter is within the interval at that level. In cases when consensus is not possible, it may be necessary to pool different estimates from a number of experts or based on a number of assessment methods. In such cases the "best estimate" itself may be an interval (such as 85 to 95) instead of a point estimate. In this case, the resulting object can be represented as a trapezoidal fuzzy number. When data are very uncertain, the range for the best estimate may coincide with the range of plausible values. In this case, the resulting object can be represented as a rectangular fuzzy number.

The IUCN classification involves arithmetic calculations, numeric magnitude comparisons and logical operations. Standard fuzzy arithmetic seems to provide a reasonable generalization for the arithmetic calculations used in the classification scheme. We used the definitions for fuzzy arithmetic operators given by Kaufmann and Gupta [11]. In this arithmetic, a fuzzy number* can be considered a set of nested intervals $[a_{li}, a_{ri}]$ at each of many levels i from zero to one representing possibility, where $a_{li} \leq a_{ri}$ for all $i \in [0,1]$, and $a_{li} \leq a_{lj}$ and $a_{rj} \leq a_{ri}$ whenever $i \leq j$. This means an ordinary scalar number is a special case of a fuzzy number for which $a_{li} = a_{ri}$ for all i. A fuzzy number is an ordinary interval if $a_{li} = a_l$ and $a_{ri} = a_r$ for all i. The fuzzy arithmetic operations can be specified as level-wise interval operations according to classical interval analysis [10] iterated over all possibility levels from zero to one. Although this is a clearly suitable generalization of the arithmetic calculations in the IUCN criteria, we believe the traditional fuzzy versions of the comparison and logical operations are inappropriate for this particular application. The following sections explain these concerns.

Magnitude comparisons

Magnitude comparisons between quantities play an important role in the IUCN criteria and we must decide on an appropriate generalization to use with fuzzy numbers. Fuzzy numbers are not naturally linearly ordered of course and, when two fuzzy numbers overlap at all, they are not directly comparable in the strict sense. As a result, there is a large literature on ranking and comparing fuzzy numbers [12–14] in which most investigators have been concerned with specifying some useful ordering that somehow generalizes the ordering of real numbers. However, many of the approaches that have been suggested are based on implicit defuzzifications or on evaluating the relative overlap of the distributions to obtain the truth or fuzzy truth of an expression like "$A \leq B$". It was not clear, however, how to correctly use such summaries in the IUCN classification scheme. It doesn't make sense, for instance, that a comparison involving an uncertain quantity whose value might be so low as to imply the species is critically endangered or might be so high as to imply lower risk should split the difference and arbitrarily assign a middle category. Yet most approaches to making numerical comparisons between fuzzy

*Some authors prefer to call such objects 'fuzzy intervals' and reserve the term 'fuzzy number' for the special case when $a_{l1}=a_{r1}$.

numbers seem to abandon the uncertainty these methods are intended to propagate. If we think that there exist true values for both A and B which we simply do not happen to know, then it doesn't make always sense to pretend such comparisons could always yield a scalar summary.

We adopted a more conservative definition based on the idea that overlapping measurement errors means the answer to a magnitude comparison should properly be "don't know" rather than any single scalar number. For example, consider the intervals $a = [1,4]$ and $b = [2,10]$. We hold that the correct value of the comparison $a \le b$ is "don't know" which we take as the vacuous logical interval [0,1]. Thus numerical magnitude comparisons between intervals are defined according to the following four rules:

$$a < b = \begin{cases} 1 & \text{if } a_r < b_l \\ 0 & \text{if } b_r \le a_l, \\ [0,1] & \text{otherwise} \end{cases}$$

$$a \le b = \begin{cases} 1 & \text{if } a_r \le b_l \\ 0 & \text{if } b_r < a_l, \\ [0,1] & \text{otherwise} \end{cases}$$

$$a > b = \begin{cases} 1 & \text{if } a_l > b_r \\ 0 & \text{if } b_l \ge a_r, \text{ and} \\ [0,1] & \text{otherwise} \end{cases}$$

$$a \ge b = \begin{cases} 1 & \text{if } a_l \ge b_r \\ 0 & \text{if } b_l > a_r. \\ [0,1] & \text{otherwise} \end{cases}$$

Fuzzy comparisons are built from the interval comparisons simply by iterating them over all possibility levels from zero to one. Thus, comparisons between fuzzy numbers typically yield h-shaped or reverse-h-shaped fuzzy numbers as results rather than single scalar values. A comparison between fuzzy numbers yields a scalar value only if the fuzzy numbers do not overlap at all, in which case the only possible values are zero or one

Logical operations

There have been many generalizations of the AND and OR operations proposed in the literature on uncertainty propagation, and fuzzy interpretations of the logical operations have been debated throughout the history of fuzzy logic [14]. Many different logical operators have been proposed for fuzzy logic, although each may be considered arbitrary in its own way. The difference among the fuzzy logical operators can be interpreted as distinct assumptions each makes about the dependence between the operands. The min and max functions originally used as fuzzy logical operators effectively assume that the operands are perfectly and positively dependent [15]. Although this seems much better than assuming independence among the quantities mentioned in the IUCN criteria (which will rarely if ever be a tenable assumption), it is still a rather strong assumption. Although it is perhaps reasonable to expect some kind of positive dependence among the components in the logical expression, it is probably not reasonable to presume they have as a relationship quite this tight.

As an inclusive compromise, we again generalize the fuzzy operators so they are conservative enough to reflect the profound uncertainty biologists typically have about such relationships. We compute *bounds* on the conjunction and disjunction without making any assumption about the dependence between the operands except that it is positive (but not necessarily perfectly so). Therefore we use the operations:

$$a \text{ AND } b = \text{env}(a \times b, \min(a,b)),$$

$$a \text{ OR } b = \text{env}(\max(a,b), 1-(1-a)\times(1-b)),$$

where

$$\min(a,b) = [\min(a_l,b_l), \min(a_r,b_r)],$$

$$\max(a,b) = [\max(a_l,b_l), \max(a_r,b_r)],$$

$$\text{env}(a,b) = [\min(a_l,b_l), \max(a_r,b_r)],$$

$$a + b = [a_l + b_l, a_r + b_r],$$

$$a - b = [a_l - b_r, a_r - b_l],$$

$$a \times b = [a_l \times b_l, a_r \times b_r],$$

This formulation posits that positive dependence is a reasonable presumption but acknowledges that the exact strength of this dependence is usually unclear because empirical information is lacking. This makes the results conservative with respect to uncertainty compared to the classical fuzzy logical operators min and max. It would also be possible to compute bounds on the conjunction and disjunction that make *no assumption whatever* about the dependence between a and b by using the classical Fréchet bounds [16,17,9]. The resulting bounds would be roughly twice as wide as those we compute.

Defuzzification

It is possible to condense the results of the calculation to a single-category classification with an additional defuzzification operation. We suggest, however, that this step is not necessary. The fuzzified classification appears to be intelligible to conservation biologists on its own terms. Preliminary feedback from users seems to suggest that biologists are comfortable with the system and appreciate the honest propagation of uncertainty in the input parameters. Defuzzification may in fact often be counterproductive in practical applications, including ranking species by their degrees of endangerment for allocating conservation resources. It obscures the true uncertainty about the classification which can be an important consideration when negotiating policy options.

On the other hand, given that one does defuzzify, it is very useful that its algorithm must be explicit. Any defuzzification step requires the analyst to explicitly specify his or her attitudes about the tradeoffs between evidence and precautions concerning endangerment. In previous analyses based on point estimates, such attitudes had always been implicit in the choice of scalar values used in the classification, but were usually ambiguous and sometimes contradictory among analyses or even with an analysis. Making these attitudes and choices explicit will help analysts and decision makers openly distinguish between what is known and what is mere opinion. Any defuzzification step requires the analyst to be explicit about formerly unacknowledged attitudes and tradeoffs about the interpretation of evidence and the prudence of precautionary classification.

A defuzzification can be accomplished by specifying three cut levels corresponding to the analyst's tolerance of scientific dispute, tolerance of risk, and attitude about burden of proof.

Dispute tolerance (DT) determines the level to use in determining the degree of threat. The bottom of the fuzzy number represents the case that includes all possibilities (DT=0). The top of the fuzzy number represents the case when dispute tolerance is highest (consensus; DT=1). Dispute tolerance represents how the person interpreting the data feels about uncertainty. If the interpreter wants to be sure of encompassing all possibilities (thus avoiding disagreements and dispute), they would use DT close to 0. If they wish to be as precise as possible and rely only on people's best judgement, they would use DT close to 1. A high DT would mean disregarding the more extreme opinions (which might lead to dispute). If the tails of the distribution seem too extreme, but the best estimates are too small to bound uncertainty, the interpreter may choose an intermediate value of, say, DT = 0.5.

The second attitude is Risk Tolerance (RT). It also ranges from 0 (risk-averse; precautionary), through 0.5 (risk-neutral), to 1 (risk-prone; evidentiary). A precautionary attitude would accept that a species is safer than endangered only if one is quite sure that it is not endangered, and RT would be closer to 0 than to 1. An evidentiary attitude would demand substantial evidence of endangerment before allowing such a classification, and RT would be closer to1. As when dealing with dispute tolerance, the interpreter may choose to take an intermediate stance with, say, RT = 0.5, or may wish to vary risk tolerance from case to case, depending on what may be lost or gained by a classification that might be either too high (risk is overestimated) or too low (risk is underestimated). Of course, mistakes of both kinds carry consequences in terms of the inefficient use of scarce conservation resources, and the possibility of inappropriate priorities for action that lead to a species' demise that might otherwise have been avoided.

We call the last requisite attitude the Burden of Proof (BP). BP also ranges from 0 to 1. A low value indicates an attitude that a taxon should be classified in the highest threat category with the range of categories (determined by the data and other attitude options). A high value indicates that the taxon should be classified in the lowest threat category within the range. BP is conceptually related to Risk tolerance, and in most cases, the same value can be used for both.

Software

The approach outlined here has been implemented in software for the Windows 95 operating system [18]. The software supports textual and graphical editing of inputs, an elaborate help and tutorial system, including several example files based on real species. It permits defuzzification when the defining attitudes about evidence and prudence are specified. It is currently under consideration for use by the IUCN. If adopted, it would be a significant example of the practical application of fuzzy methods in an important branch of applied science.

Conclusions

The straightforward use of intervals and triangular and trapezoidal fuzzy numbers in place of scalar estimates can transform the widely used IUCN criteria into a system that can acknowledge measurement error and scientific uncertainty. The complex criteria involve arithmetic calculations, magnitude comparisons and logical operations. Standard fuzzy arithmetic sensu Kaufmann and Gupta were used in place of ordinary arithmetic within the IUCN criteria, but the traditionally defined fuzzy comparisons seemed inappropriate and were replaced by more conservative formulations. Likewise, the traditional fuzzy logic operations were considered implausible because of the underlying biological mechanisms involved. Instead, logical conjunctions and disjunctions (ANDs and ORs) are computed under the assumption of positive associations between their operands.

The result of the fuzzified classification scheme would in general be a *set* of categories. For instance, a taxon may be known to be either endangered or critically endangered (which we might denote by [EN, CR]).

When data are so deficient as to preclude any assignment at all, the result would naturally be the range [LR, CR], i.e., any status between the non-threatened "lower risk" category and critically endangered. Biologists seem to be comfortable with such non-specific designations, and recognize that it arises from the measurement error in the input description of the taxon. They tend to appreciate a method that handles uncertainty in a reliable way.

It is possible of course to defuzzify the result to obtain a single category for each taxon, but this generally requires the specification of arbitrary thresholds and tradeoffs. One must specify, for instance, one's attitudes about evidence. One must decide whether a taxon is to be considered threatened until proven otherwise, or considered non-threatened until proven otherwise. A particularly useful feature of an approach based on fuzzy numbers is that a defuzzification step requires the analyst to explicitly specify his or her attitudes about the tradeoffs between evidence and precautions concerning endangerment. In previous analyses based on point estimates, such attitudes had always been implicit in the choice of scalar values used in the classification, but were usually ambiguous and sometimes contradictory. Making these attitudes and choices explicit will help analysts and decision makers openly distinguish between what is known and what is mere opinion.

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